Mean Dimension Expansive Dynamical Systems

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Celebrating Maria José Pacifico's 70th birthday

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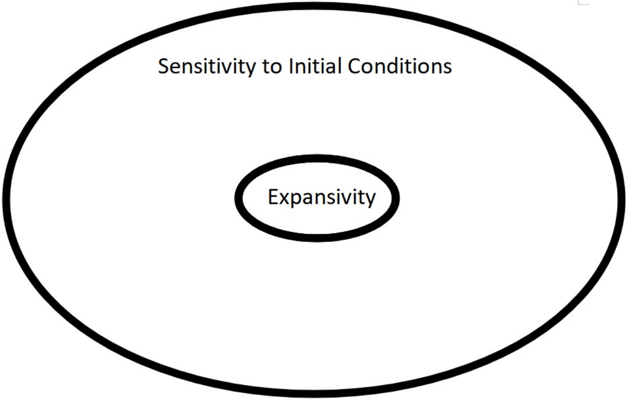
Motivation



Expansivity

Definition 1 (Utz; PAMS 1950)

We shall say that f is expansive if there is $\delta > 0$ such that

$$\Gamma_\delta(x) = W^s_\delta(x) \cap W^u_\delta(x) = \{x\}$$

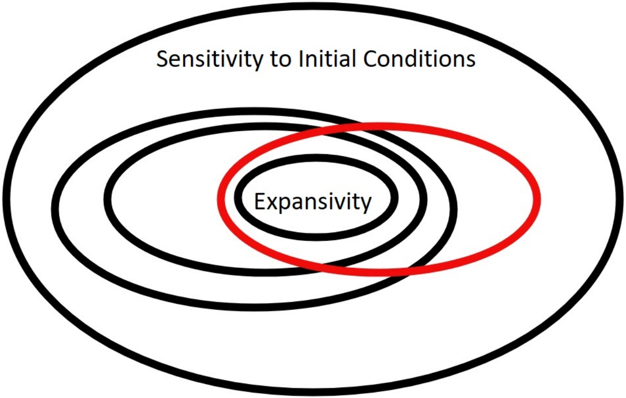
for each $x \in X$.

Infinite dimension?

Theorem 2 (Mané; 1979)

If $f : X \to X$ is an expansive homeomorphism, then the topological dimension of X is finite.

Motivation



CW-Expansive

Definition 3 (Kato; CJM 1993)

We shall say that f is Continuum Wise expansive if there is $\delta > 0$ such that if $A \subset X$ is a non-trivial continuum, then there is $n \in \mathbb{Z}$ such that $\operatorname{diam}(f^n(A)) > \delta$.

Infinite Dimension?

Theorem 4 (Kato; CJM 1993)

If $f : X \to X$ is an CW-expansive homeomorphism, then the topological dimension of X is finite.

Topological Entropy

Definition 5

$E \subset K \subset X$ is an (n, ϵ)-*separated set* if

$$\sup_{|i|<n} d(f^i(x), f^i(y)) > \epsilon, \quad \forall x, y \in E.$$

Let $s(n, \epsilon, K)$ be the maximal cardinality of an (n, ϵ)-separated set in K.

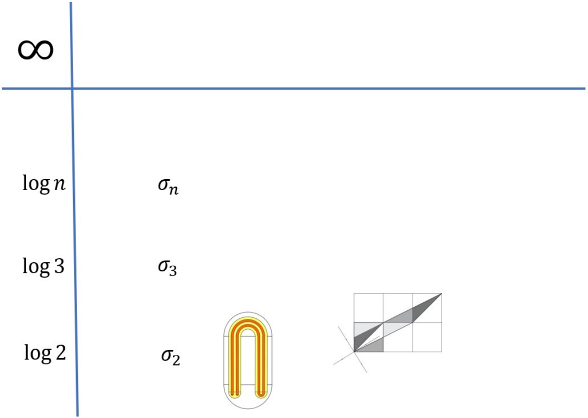
Topological Entropy

Definition 6

The *Topological Entropy* of f in $K \subset X$ is obtained as

$$h(f,K) = \lim_{\epsilon \to 0} \limsup_{n \to \infty} \frac{1}{n} \log s(n,\epsilon,K).$$

Topological Entropy



Mean Dimension [Gromov, 1999]

Definition 7

Let α be an open cover of X. Define the *order* of α by

$$ord(\alpha) = \max_{x \in X} \sum_{U \in \alpha} 1_U(x) - 1.$$

and

$$D(\alpha) = \min_{\beta \succ \alpha} ord(\beta)$$

.

Mean Dimension [Gromov, 1999]

Definition 8

Let α be an open cover of X. Define

$$\alpha^n = \bigvee_{i=0}^{n-1} f^{-i}\alpha.$$

Finally, define the **Mean Dimension** of f by

$$mdim(X, f) = \sup_{\alpha} \lim_{n\to\infty} \frac{1}{n} D(\alpha^n).$$

Mean Dimension [Gromov, 1999]

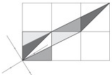
Proposition (Lindenstrauss and Weiss, 2000)

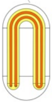
Let Σ_d *be the shift on* $([0,1]^d)^{\mathbb{Z}}$. *Then*

$$mdim(([0,1]^d)^{\mathbb{Z}}, \Sigma_d) = d.$$

Mean Dimension [Gromov, 1999]

Topological Entropy		*Mean Dimension*
$+\infty$	$(([0,1]^d)^{\mathbb{Z}}, \Sigma_d)$	d
	$(([0,1]^2)^{\mathbb{Z}}, \Sigma_2)$	2
	$([0,1]^{\mathbb{Z}}, \Sigma_1)$	1
$\log n$	$(\{0, \dots, n\}^{\mathbb{Z}}, \sigma_n)$	0
$\log 3$	$(\{0,1,2\}^{\mathbb{Z}}, \sigma_3)$	
$\log 2$	$(\{0,1\}^{\mathbb{Z}}, \sigma_2)$	





Entropy expansive

Definition 9 (Bowen; TAMS 1972)

We shall say that f is Entropy expansive if there is $\delta > 0$ such that

$$h(\Gamma_\delta(x)) = 0$$

for each $x \in X$.

Mean Dimension Expansive

Definition 10

We shall say that f is Mean Dimension Expansive if there is $\delta > 0$ such that

$$mdim(\Gamma_\delta(x)) = 0$$

for each $x \in X$.

Mean Dimension Expansive

Definition 11

We shall say that f is Mean Dimension Expansive if there is $\delta > 0$ such that

$$mdim(\Gamma_\delta(x)) = 0$$

for each $x \in X$.

Metric Mean Dimension [Lindenstrauss and Weiss, 2000]

Definition 12

The *Metric Mean Dimension* of (f, d) in $K \subset X$ is obtained as

$$mdim_M(f, K, d) = \liminf_{\epsilon \to 0} \frac{1}{|\log \epsilon|} \limsup_{n \to \infty} \frac{1}{n} \log s(n, \epsilon, K).$$

And the *Metric Mean Dimension* of f in K by

$$mdim_M(f, K) = \inf_d mdim_M(f, K, d)$$

Metric Mean Dimension [Lindenstrauss and Weiss, 2000]

Theorem 13 (Lindenstrauss and Weiss, 2000)

For any metric d on X we have

$$mdim(f) \leq mdim_M(f, d)$$

Theorem 14 (Lindenstrauss, 1999)

If f is an extension of a minimal system, then there is a metric d such that

$$mdim(f) = mdim_M(f, d)$$

Metric Mean Dimension Expansive

Definition 15

We shall say that f is Metric Mean Dimension Expansive if there is $\delta > 0$ such that

$$mdim_M(\Gamma_\delta(x)) = 0$$

for each $x \in X$.

Mean Dimension Expansive

- $\{f_n\}$ with $h(f_n) \to \infty$. The product with the Tychonoff topology
- $(([0,1]^d)^{\mathbb{Z}}, \Sigma_d)$ are not Metric mean dimension expansive

Thank you!

Thank you for your attention!

Parabéns e felicidades Zezé!