

On continuum-wise minimality

(M, d) compact metric space, $f: M \rightarrow M$ homeomorphism.

$\dim =$ topological dimension

$X \subset M$ closed, $\dim X = 0$ iff X is totally disconnected

X is a continuum if it is closed and connected.

Dynamical ball: $\varepsilon > 0$, $\Gamma_\varepsilon(x) = \{y \in M : d(f^i x, f^i y) \leq \varepsilon \ \forall i \in \mathbb{Z}\}$
 $x \in M$

f is expansive if $\exists \varepsilon > 0 / \Gamma_\varepsilon(x)$ is trivial $\forall x \in M$
 $\Gamma_\varepsilon(x) = \{x\}$

f is minimal if $X \subset M, X \neq M$ closed invariant
 $\Rightarrow X$ is trivial
 $X = \emptyset$.

f is ω -expansive if $\exists \varepsilon > 0 / \dim(\Gamma_\varepsilon(x)) = 0 \ \forall x \in M \iff$

f is ω -minimal (Kato) if $X \subset M, X \neq M$ closed invariant
 $\Rightarrow \dim X = 0$ or $X = \emptyset$.

We assume $\dim M > 0$

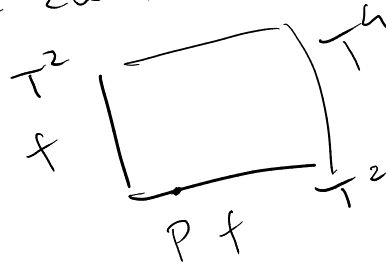
Prop minimal $\Rightarrow \omega$ -minimal \Rightarrow transitive.

Prop ω -minimal $\iff M \setminus \text{Trans}(f)$
 is totally disconnected.

Examples Pseudo-Anosov diffeos are ω -minimal (Kato).

Rank Anosov may not be ω -minimal.

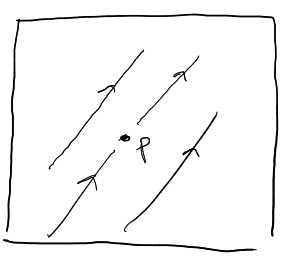
codim 1 $\Rightarrow \omega$ -min?
 $T^3?$



$P \in \text{Fix}(f)$
 $P \times T^2$
 $\dim \rightarrow 2$

Prop (Kato) cw-exp + cw-min \Rightarrow transitive...
 • $\text{clos}(UX) = M$: chaotic
 $x \in M$
 minimal
 • sensitive

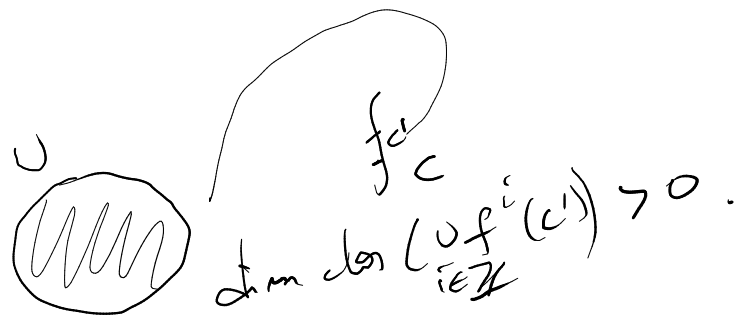
Example



#2 ϕ irrational flow with p singular.

$f = \phi_1$ is cw-min
 $\text{Trans}(f) = M \setminus \{p\}$

Prop M locally connected, $\dim M > 0$
 f cw-minimal \Rightarrow minimal or sensitive.

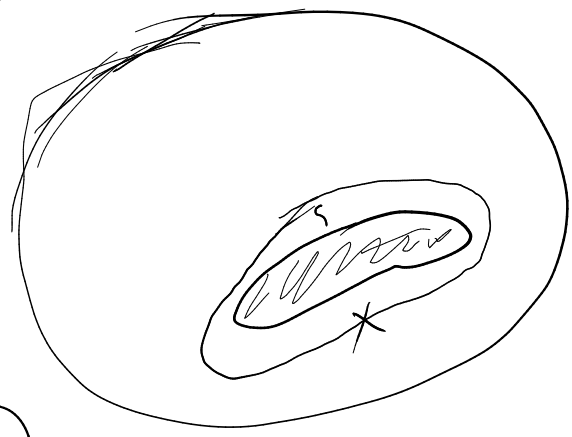


$\cup_{i \in \mathbb{Z}} f^i(U) \cap C$ dense by subset.

idea of proof

if f is not min.

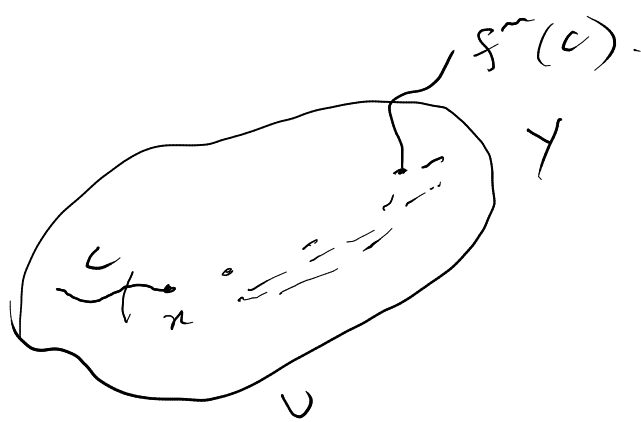
M $\dim X = 0$.



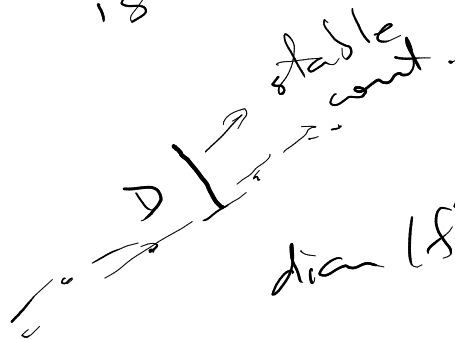
cw-isolated
 \hookrightarrow



\Rightarrow \exists isolated set.



$\dim Y = 0$
isolated.



stable
cont.
 $\text{diam}(f^n D) \rightarrow 0$
 $n \rightarrow \infty$.

this and transitivity.
 \Rightarrow sensitivity. \square



f^2

Attractor

\Rightarrow cw-min
cw-max

Not mixing

